the characteristic scale is provided by the ion range $L_x \sim L_i$, but behind the SW front one should use gas dynamics in view of the high density ($h \sim 10^5$, $L_x \gg L_i$), i.e., the discrete structure is unimportant, and the process is of macroscopic character.

It is also necessary to examine the spectral treatment, since the formation of the T corona is accompanied by the production of quanta in the x-ray range.

A description has been given [7] of the mathematical formulation considered here. We merely note that introducing the temperature $T_f$ implies that a Planck spectrum is applicable.

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# POSSIBILITY OF THE DEVELOPMENT OF OSCILLATIONS DURING THE HEATING OF A TRANSPARENT SOLID DIELECTRIC BY OPTICAL RADIATION

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Interest in the study of breakdown mechanisms in transparent solids has not diminished in the last two decades. New ideas have appeared regarding the breakdown of media both with a pronounced band structure of the energy levels and amorphous materials with broadened bands. Under certain conditions the difference between these two becomes insignificant. Beginning in the 1970's concepts of the breakdown of transparent solids by the heating of the medium near strongly absorbing admixtures or inhomogeneities were developed [1, 2]. Concepts of linear and nonlinear heating mechanisms and breakdown of materials were developed [1-4]. The nonlinear model of the development of breakdown leads to good qualitative agreement of theory and experiment in a rather large number of cases. However, recent experiments [5] show that in glassy systems the behavior of the breakdown threshold as a function of the freedom of the material from admixtures does not agree with ideas corresponding to the nonlinear model of the development of breakdown at trace impurities. Ideas have been expressed on the effect of fluctuations of the microstructure of a material on the corresponding behavior of the characteristics of the breakdown of the medium. All this indicates the need for further study of the processes occurring during the heating by optical radiation of a material which does not contain an appreciable number of microinclusions. Papers have appeared on the effect of the temperature dependence of the thermal conductivity on the overall development of the heating of the material. The results indicate the formation of a region in which the thermal conductivity is large, and the temperature and the characteristics related to it are practically constant over the coordinate [6]. This result was obtained under certain assumptions, and in particular without taking account of the possibility of the interaction of relaxations through thermal and elastic channels. In certain regions of the medium the heat-transfer rate can become comparable with the rate of propagation of elastic perturbations. This shows that

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coupling may occur as a result of nearly the same rates of change of the temperature and elastic stress distributions, and this may substantially affect the overall pattern of the breakdown of the material.

Because of the complexity of the problem, we do not attempt to solve it in its most general form, but consider a rather simple one-dimensional case. We investigate the possibility of the development of temperature and elastic stress oscillations in a domain in which as a result of prior radiation heating the temperature is rather high and practically independent of the coordinate. We seek the behavior of the temperature and elastic stress distributions in an unbounded homogeneous region. A uniform constant light flux is propagating in this medium in the positive direction of the x axis. In this case the heat-conduction equation has the form

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\varkappa \frac{\partial T}{\partial x}\right) + I\beta, \tag{1}$$

where $\rho$ is the density of the medium, c is the specific heat, T is the temperature, $\varkappa$ is the thermal conductivity, I is the intensity of the light flux, and $\beta$ is the absorption coefficient. We assume that $\varkappa$ and $\beta$ in Eq. (1) have the same temperature dependence. In solving Eq. (1) we assume that over distances considered in the problem I varies relatively slowly with the coordinate. This corresponds to conditions appreciably different from breakdown (over characteristic distances the change in intensity due to absorption is relatively small). The temperature dependence of $\beta$ is similar to that assumed in nonlinear models [7]:

$$\beta = \beta_1 \exp\left\{-\frac{E_0 + \gamma\sigma_{xx}}{k_* T}\right\}, \tag{2}$$

where $E_0$ is proportional to the width of the forbidden band (the characteristic energy), $\sigma_{xx}$ is a component of the elastic stress tensor, and $\gamma$ is a proportionality factor. The presence of $\sigma_{xx}$ in (2) corresponds to taking account of the dependence of the width of the forbidden band of the material on the elastic stresses developed in the medium. The elasticity theory equations [8] lead to

$$\rho \frac{\partial^2 u}{\partial t^2} = -K\alpha \frac{\partial T}{\partial x} + \left(K + \frac{4\mu}{3}\right)\frac{\partial^2 u}{\partial x^2}, \tag{3}$$

where $u = u(x, t)$ is the displacement of points of the medium, K and $\mu$ are expressed in terms of Young's modulus E and Poisson's ratio $\sigma$ by the formulas $K = E/3(1 - 2\sigma)$ and $\mu = E/2 \cdot (1 + \sigma)$, and $\alpha$ is the volume coefficient of expansion. For $\sigma_{xx}$ we have

$$\sigma_{xx} = -K\alpha(T - T_0) + \left(K + \frac{4\mu}{3}\right)\frac{\partial u}{\partial x}. \tag{4}$$

We linearize Eqs. (1) and (3) with respect to the initial uniform distribution, and then determine the possibility of the development of oscillations in the system.

We perform the linearization by expanding the variables at a certain instant in terms of $T - T_0$, the deviation of the temperature from $T_0$. The zero level of elastic stress corresponds to $T_0$. The initial distribution of all quantities is completely uniform and independent of x. Therefore, the spatial derivatives are first-order quantities. In view of this the spatial dependence of $\varkappa$ can be neglected, since it is of second order. We write the absorption coefficient in the form

$$\beta = \beta_0 \exp\left\{-\frac{E_0 + \gamma\sigma_{xx}}{k_* T} + \frac{E_0}{k_* T_0}\right\}, \tag{5}$$

which corresponds to the definition of $\beta_0$ as the absorption coefficient at $T = T_0$. The expansion of (5) leads to

$$\beta = \beta_0\left\{1 + \frac{(T - T_0)}{k_* T_0^2}(\gamma K\alpha T_0 + E_0) - \frac{\gamma}{k_* T_0}\left(K + \frac{4\mu}{3}\right)\frac{\partial u}{\partial x}\right\}. \tag{6}$$

The value of $\sigma_{xx}$ given by Eq. (4) was used in (6). Introducing the variable $\theta$ by the formula

$$T = T_0 + T_0 t/\tau_1 + \theta \tag{7}$$

and the constants

$$M = E_0/(k_* T_0), \quad m = \gamma K\alpha T_0/(k_* T_0),$$
$$\chi = \frac{\varkappa_0}{\rho c}, \quad \tau_1 = \frac{\rho c T_0}{I\beta_0}, \quad c_*^2 = \frac{K + \frac{4\mu}{3}}{\rho},$$

and substituting (7) into (1) and (3), we obtain

$$\frac{\partial\theta}{\partial t}=\chi\frac{\partial^2\theta}{\partial x^2}+\frac{M+m}{\tau_1}\theta-\left(\frac{c_*^2\rho\gamma}{\tau_1 k_*}\right)\frac{\partial u}{\partial x},\qquad(8)$$
$$\frac{\partial^2 u}{\partial t^2}=-\left(\frac{K\alpha}{\rho}\right)\frac{\partial\theta}{\partial x}+c_*^2\frac{\partial^2 u}{\partial x^2}.$$

In writing the right-hand side of the heat-conduction equation in (8) we have neglected $T_1 = T_0 t/\tau_1$ in comparison with $T_2 = \theta$. This implies that $T_2$ may be appreciably larger than $T_1$ during the time interval considered, and $T_1$ at all times is intrinsically small. At the same time there are no restrictions on the ratio of the rates of change of $T_1$ and $T_2$. For the expansion to be applicable, the conditions $t \ll \tau_1$ and $\theta \ll T_0$ must also be satisfied.

We seek the solution of Eq. (8) in the form

$$\theta=\theta_0\exp\{i(kx-\omega t)\},\ u=u_0\exp\{i(kx-\omega t)\}.\qquad(9)$$

After substituting (9) into (8), we obtain the dispersion relation in the form

$$(\omega^2-c_*^2k^2)\left(\omega+i\left(\chi k^2-\frac{M+m}{\tau_1}\right)\right)=i\frac{k^2c_*^2m}{\tau_1}.\qquad(10)$$

An important consequence of Eq. (10) is the fact that complex values of the frequency may exist, which means that oscillations of the indicated type can increase exponentially.

Equation (10) is cubic in $\omega$, and in general has three different solutions. We analyze the behavior of the solutions under the assumption that the thermoelastic stresses have a small effect on the parameters of the medium, i.e., $m/M \ll 1$. In doing that we take the following values of the parameters: $\varkappa_0 = 10\ \mathrm{W/m\cdot{}^\circ K}$, $\rho = 3\cdot10^3\ \mathrm{kg/m^3}$, $c = 1.3\cdot10^3\ \mathrm{J/kg\cdot{}^\circ K}$, $I = 10^{13}\ \mathrm{W/m^2}$, $\beta_0 = 1\ \mathrm{m^{-1}}$, $T_0 = 1000^\circ\mathrm{K}$, $E_0 = 60k_*T_0$, $\gamma = 2\cdot10^{-3}k_*T_0/p_0$, $p_0 = 10^5$ Pa, $\alpha = 8\cdot10^{-3}/T_0$, $K = 4.2\cdot10^{10}$ Pa, $E = 7\cdot10^{10}$ Pa, $\sigma = 0.22$, and $\mu = 2.9\cdot10^{10}$ Pa. The elastic parameters, density, and specific heat correspond to a fused quartz glass medium; the absorption coefficient and the thermal conductivity were estimated by taking account of prior heating of the medium; the intensity was taken equal to typical values for experiments on the breakdown of solid dielectrics. For the assumed values of the parameters, $m/M \sim 0.1$ and $\tau_1 \approx 4\cdot10^{-4}$ sec.

In the limiting case $m/M = 0$, Eq. (10) leads to two types of solutions with $\omega_{1,2}^{(0)} = \pm c_*k$ (acoustic oscillations) and $\omega_3^{(0)} = i(M/\tau_1 - \chi k^2)$ (entropy mode). Oscillations with $\omega_3$ are a typical mode of thermal fracture. The corresponding solution increases exponentially for small values of k, and is damped for large k. For $k = 0$ the characteristic rise time of this mode is $\tau_2 = \tau_1/M$. The ratio $t/\tau_2$ determines whether the condition $\theta \ll T_0$ mentioned above is satisfied. If we assume that $t_*/\tau \approx 1$ is the condition for the threshold of breakdown of the material by laser radiation ($t_*$ is the pulse duration), we obtain for the threshold intensity

$$I_*\approx\frac{\rho cT_0}{\beta_0 t_*}\left(\frac{k_*T_0}{E_0}\right).\qquad(11)$$

In using (11) it must be kept in mind that $T_0$ is the temperature at which the rate of heat transfer in the medium becomes comparable with the rate of transfer of elastic perturbations, and $E_0$ is the value of the characteristic energy in (2) at $T = T_0$.

The threshold value of $k_1^2$ for which $\omega_3^{(0)} = 0$ is $M/\tau_1\chi$. For the values of the parameters chosen, $k_1 \approx 2\cdot10^5\ \mathrm{m^{-1}}$ and $\lambda_1 \approx 3\cdot10^{-5}$ m. The acoustic modes for $m/M = 0$ are not coupled to the entropy mode, and do not increase.

The behavior of the solutions of Eq. (10) for $m/M \neq 0$ corresponding to $\omega_{1,2}^{(0)}$ and $\omega_3^{(0)}$ can be determined to first-order accuracy in m/M. Solving (10) by iteration and stopping at an appropriate step, we obtain

$$\omega_{1,2}^{(1)}=\omega_{1,2}^{(0)}+\Delta\omega_{1,2}(m),\quad \omega_3^{(1)}=\omega_3^{(0)}+\Delta\omega_3(m),$$
$$\Delta\omega_{1,2}(m)=\left[-m\omega_{1,2}^{(0)}\left(\frac{M}{\tau_1}-\chi k^2\right)+imk^2c_*^2\right]\left\{2\tau_1\left[k^2c_*^2+\left(\frac{M}{\tau_1}-\chi k^2\right)^2\right]\right\}^{-1},\qquad(12)$$
$$\Delta\omega_3(m)=i\frac{m}{\tau_1}\left(1-k^2c_*^2\left[k^2c_*^2+\left(\frac{M}{\tau_1}-\chi k^2\right)^2\right]^{-1}\right).$$

The form of $\Delta\omega(m)$ in (12) indicates an interaction of the acoustic and entropy modes for $k \neq 0$ during the absorption of optical radiation. For $k = 0$ there is no such interaction,

acoustic perturbations do not increase, and the rate of growth of the amplitude of the thermal perturbation is maximum. For $k \neq 0$ there is a relative lowering of the rate of growth of the amplitude of thermal perturbations and a corresponding increase in the rate of growth of the amplitude of acoustic perturbations, which indicates the transfer of part of the absorbed energy to acoustic oscillations. The growth rate of the acoustic mode as a function of k is maximum for

$$k_m^2 = \frac{M}{\chi\tau_1}, \tag{13}$$

which corresponds to $k_m \approx 2 \cdot 10^5$ m$^{-1}$, $\lambda_m \approx 3 \cdot 10^{-5}$ m, and $\omega_m \approx k_m c_* \approx 10^9$ sec$^{-1}$. In an experiment to which the theory developed is applicable, one can expect the appearance of acoustic oscillations with parameters close to the values given above.

A complete determination of the character of the redistribution of absorbed energy requires taking account of the variation of the frequency of acoustic oscillations with k in accordance with (12).

A condition similar to (11) can be written for the threshold intensity when $m/M \neq 0$. For $k = 0$ it has the form

$$I_* \approx \frac{\rho c T_0}{\beta_0 t_*} \left(\frac{k_* T_0}{E_0}\right)\left[1 + \frac{\gamma K \alpha T_0}{E_0}\right]^{-1}. \tag{14}$$

According to (14) one can expect a weak dependence of the threshold intensity on $\gamma$, which determines the effect of thermoelastic stresses on the characteristic energy. An experimental test of (14) does not seem simple, but it is of great interest. An experimental test of (13) and its consequences is more practicable.

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